Solutions

4.1-4.2: Matrix Addition, Scalar Multiplication and Matrix Multiplication

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Example 1. Let

$$A = \begin{bmatrix} 7 & 9 & x \\ 0 & -1 & y+1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 & 9 & 0 \\ 0 & -1 & 11 \end{bmatrix}.$$

Find x and y so that $A = B$. This gives 6 equation $7=7, 9=9, x=0$, $0=0, -1=-1, y+1=11$ $\Rightarrow$ $x=0$, $y=10$

Example 2. Find the desired quantities. Each gives 6 equations below.

(a) $$\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 9 & -5 \\ 0 & 13 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 2+9 & -3+(-5) \\ 1+0 & 0+13 \\ -1+(-1) & 3+3 \end{bmatrix} = \begin{bmatrix} 11 & -8 \\ 1 & 13 \\ -2 & 6 \end{bmatrix}$$

(b) $$\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ -1 & 3 \end{bmatrix} - \begin{bmatrix} 9 & -5 \\ 0 & 13 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 2-9 & -3-(-5) \\ 1-0 & 0-13 \\ -1-(-1) & 3-3 \end{bmatrix} = \begin{bmatrix} -7 & 2 \\ 1 & -13 \\ 0 & 0 \end{bmatrix}$$

Remark 1. Two matrices can be added or subtracted only when they have the same dimensions. In the above example, both matrices are 3×2 and therefore the sum and difference is defined.

Exercise 1. Let

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 5 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 & -1 \\ 5 & -6 & 0 \end{bmatrix}, \text{ and } C = \begin{bmatrix} x & y & w \\ z & t+1 & 3 \end{bmatrix}.$$

Evaluate the following: 4A, xB, and A+3C.

$$4A = \begin{bmatrix} 8 & -4 & 0 \\ 12 & 20 & -12 \end{bmatrix}$$

$$xB = \begin{bmatrix} x & 3x & -x \\ 5x & -6x & 0 \end{bmatrix}$$

$$A + 3C = A + \begin{bmatrix} 3x & 3y & 3w \\ 3z & 3(t+1) & 9 \end{bmatrix} = \begin{bmatrix} 2+3x & 3y-1 & 3w \\ 3z+3 & 3t+8 & 6 \end{bmatrix}$$

The transpose flips columns and rows.

Example 3. Find the transpose of matrix A in exercise 1.

$A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 5 & -3 \end{bmatrix} \qquad A^T = \begin{bmatrix} 2 & 3 \\ -1 & 5 \\ 0 & -3 \end{bmatrix}$

Example 4. Suppose we download 3 movies at \$10 each and 5 albums at \$8 each. We will find the total amount of money spent using matrix multiplication.

Price $= \begin{bmatrix} 10 & 8 \end{bmatrix}$ quantity $= \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ Cost $= P \cdot q = \begin{bmatrix} 10 & 8 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$= \begin{bmatrix} 10 \cdot 3 + 8 \cdot 5 \end{bmatrix}$

$= \begin{bmatrix} 70 \end{bmatrix}$ a 1×1-matrix

Example 5. Find the following products.

(a) $\begin{bmatrix} 2 & 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & -8 \\ 1 & -6 & 0 \\ 0 & 5 & 2 \\ -3 & 8 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 21 & -15 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 1 \cdot 3 + (-1) \cdot 5 & 1 \cdot 0 + (-1) \cdot (-1) \\ 0 \cdot 3 + 2 \cdot 5 & 0 \cdot 0 + 2 \cdot (-1) \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 10 & -2 \end{bmatrix}$

$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} \text{———} \\ \text{———} \\ \text{———} \end{bmatrix} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} | & | & | \\ | & | & | \\ | & | & | \end{bmatrix} \end{array} = \begin{bmatrix} R_1C_1 & R_1C_2 & R_1C_3 \\ R_2C_1 & R_2C_2 & R_2C_3 \\ R_3C_1 & R_3C_2 & R_3C_3 \end{bmatrix}$

Remark 2. The product of two matrices is only defined if they number of rows in the left matrix is equal to the number of columns in the right matrix. In this case, if A is a $k \times l$-matrix and B is a $l \times n$-matrix then the product AB is a $k \times n$-matrix.

Exercise 2. Let $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 \\ 5 & -1 \end{bmatrix}$. Find AB and BA to show that matrix multiplication does not commute.

$AB = \begin{bmatrix} -2 & 1 \\ 10 & -2 \end{bmatrix}$

$BA = \begin{bmatrix} 3 & -3 \\ 5 & -7 \end{bmatrix}$ Clearly $AB \neq BA$ so matrix multiplication does not commute.